Leibniz Universität Hannover

Mathematic and Physic (Admission Year 2008)

Exam: Part A1 „ Mathematic “

All answers are to be reasoned!

Allowed materials:

- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time: Part A1 and A2 together 40 Minutes

4 Questions (Part A1)

Name:.. **Register Nr.** : ....

Hints:

- Write on all pages containing answer sheets, your name and register number.
- All printed question pages should be given back completely.

By corrector only:

Question Nr.	Point(Mark)	Corrector	Exam Director
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Mathematics

Problem 1) determine the eigenvalues of the matrix

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix},$$

Where one of the eigenvalues of A is a small integer. Moreover determine at least one of the eigenvectors.

Problem 2) Consider a circular cylinder of radius r and height h. Compute the total surface area in terms of r and h, and determine the dimensions to minimize the total surface area when the volume of the cylinder is constant.

Problem 3) Solve the differential equation

$$1 - y^2 - x\,\frac{dy}{dx} = 0$$

With initial condition $y(1) = 1/2,$

Hint: $\displaystyle\int \frac{d\xi}{1-\xi^2} = \ln\left(\sqrt{\frac{1+\xi}{1-\xi}}\right) + C$

Problem 4) Give the condition under which the vector field

$$\vec{v}\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$

possesses a (scalar) potential function $\varphi\colon \mathbb{R}^3 \longrightarrow \mathbb{R}$ (such a vector field is called also conservative). Determine if the vector field

$$\vec{v} := (2xy + z^3, x^2, 3xz^3)$$

is conservative or no

Mathematic and Physics (Admission Year 2008)

Exam: Part A2 „ Physics “

All answers are to be reasoned!

Allowed materials:
- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time: Part A1 and A2 together 40 Minutes

4 Questions (Part A2)

Name:………………………………………………… **Register Nr.** : ……………………….

Hints:

- Write on all pages containing answer sheets, your name and register number.
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Physics

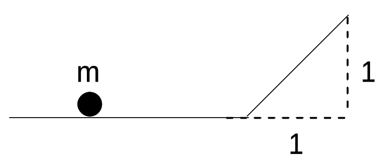
1) A particle of mass m=1kg, which is under the influence of earth's gravity, moves on a frictionless smooth surface. This particle continues its movement on an inclined plane with incline angle 45^o and $\sqrt{2}$ meter length.

When the particle leaves the smooth surface, its velocity is $v_0 = \sqrt{2} m/s$.

a) What is the distance between the stop position and the edge of the inclined plane.

(in the stop position, the particle can not go up more.)

(For the magnitude of the free-fall acceleration, use $g=10m/s^2$.)



2) Consider a wood cube with height, breadth and length as follows:

H=height= 140 mm , B=breadth= 180 mm , L=length= 200 mm

The cube is floating on water (having the density of $\rho_w = 1\times10^3\ kg/m^3$) and is partially submerged in depth h=80 mm.

A) Calculate the magnitude of the pressure exerted by the water on the floor of the cube.
B) B) Illustrate the status of the cube in a simple figure.

3) An object is placed g=65cm in front of a concave mirror. The mirror has the radius of curvature r=25 cm.

A) Calculate the distance between the image point and the mirror.

B) Illustrate the configuration of the object, image and mirror in a simple figure.

4) An electric charge accelerates uniformly in a constant electrical field. Despite this fact, charges (electrons) in a conductor wire move with an approximately constant velocity when a potential difference (and so, an electric filed) is applied across the conductor.

A) Explain this different behavior of charges.

B) Explain about the electrical conductivity of pure semiconductors nears absolute zero. (Explain: How does it change versus temperature and why?)

Leibniz Universität Hannover

Electric Circuits and
Principles of Electrotechnic (Admission Year 2008)

Exam:Part B „ Electric Circuits and Principles of Electrotechnic “

All answers are to be reasoned!

Allowed materials:

- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time: Part B 40 minutes

4 Questions (Part B)

Name:... **Register Nr.** : ....

Hints:

- Write on all pages containing answer sheets, your name and register number.
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By corrector only:

Question Nr.	Point(Mark)	Corrector	Exam Director
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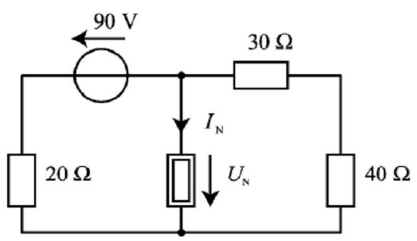
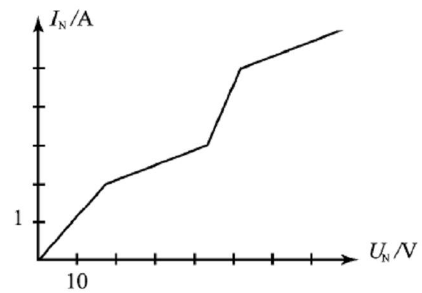
Electric Circuits (page 1/2)

1. Nonlinear resistance

The following network with a nonlinear resistance is given:

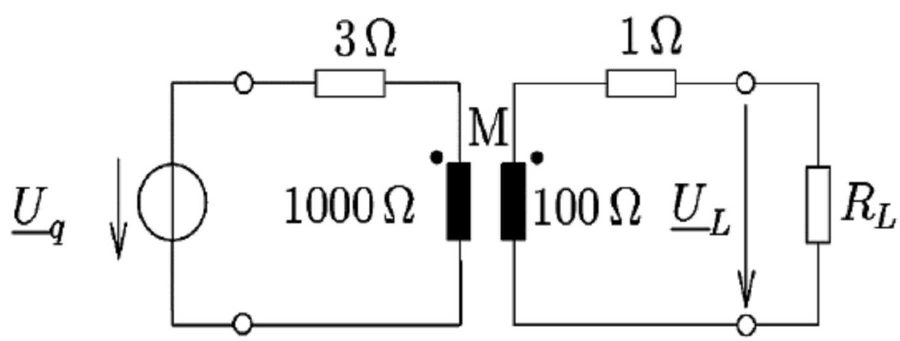




How large is the current of this resistance?

2. Network with transformer

The following harmonic excited transformer network is given: ($\omega M = 300\Omega$)



The resistive load R_L=50 Ohm, Should be supplied at the secondary side of transformer with a voltage U_L=230 V. Calculate the effective value of voltage Uq.

Electric Circuits (page 2/2)

3 Switching transient

Consider the Circuit of figure 1, in which the switch 'S' is closed at t=0. The capacitor C has been charged up to the voltage $u_C(0)=U_q$ before the switch 'S' is closed. The source current is given as: $i_q(t) = I_q e^{\alpha t} \quad \alpha > 0$

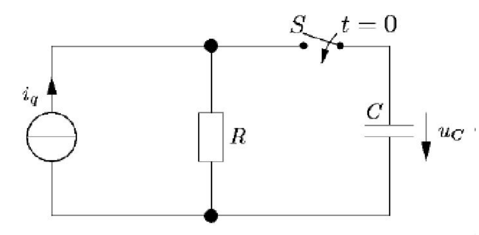


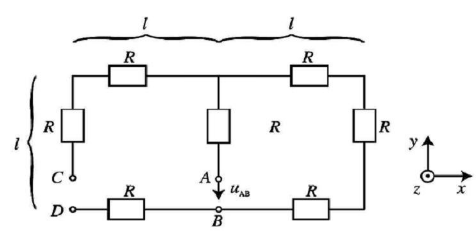
Figure 1: Network with a storage capacitor

Find a parametric expression for Iq which results in no switching transient at t=0.

4. Moving circuit in magnetic field

The resistance loops, shown in the following figure, moves with the speed of $\vec{v} = 2\,\vec{a}_x \ \frac{m}{s}$ in the direction of x through a homogeneous, time variant magnetic field $\vec{B}(t) = 0.3t\vec{a}_z \ \frac{T}{s}$.

Given: $l = 10\,cm$ and $R = 100\,\Omega$



a) If the terminals C and D are open: Calculate the voltage u_{AB}.at t=1 s.

b) Now the terminals C and D became closed: Calculate the voltage u_{AB} at t=2 s.

Signals / Systems and
Linear Systems and Control (Admission Year 2008)

Exam: Part C1 „ Signals/Systems “

All answers are to be reasoned!

Allowed materials:

- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time: Part C1 and Part c2 together 40 minutes

4 Questions (Part C1)

Name:... **Register Nr.** : ....

Hints:

- Write on all pages containing answer sheets, your name and register number.
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Signals and systems

Question 1: A linear time-invariant system (LTI) with the following definition is given:

$f(t) \rightarrow g(t) = f(t - t_0)$. the function f(t) has the Fourier transform F(jω).

1.1. Find the impulse response h(t) of the system.
1.2. Determine the Fourier Transform G(jω) of g(t) as a function of F(jω)

Consider the input signal $f(t)=j\sin(\omega_0 t)$.

1.3. Compute and plot the Fourier Transform F(jω) of f(t) .

Question 2: Consider a discrete-time system {x(n)} à {y(n)} with transfer function H(z) :

$$H(z) = \frac{2 - z^{-1}}{1 - z^{-1}}$$

2.1 Determine the difference equation of system as a function of x(n) and y(n).

2.2 Calculate the impulse response of this system

2.3 Is this system stable? (Demonstrate your answer)

Question 3: Suppose that f(t) is a periodic time function with period t_0

$f(t)=f(t+t_0)$

and $f(t)=t$ for $0<t<t_0$

3.1. Plot f(t) for $-t_0 < t < 2t_0$.

3.2. Give a general formula for the complex Fourier series coefficients representation C_n of f(t).

3.3. Calculate the Fourier series coefficients of f(t) for $n \neq 0$.

Hint: $\int x e^{ax} dx = \frac{1}{a^2}(ax-1)e^{ax}$.

Question 4: suppose a linear continues time-invariant system f(t) à g(t) with impulse response:

$$h(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & else \end{cases}$$

The input signal is: $f(t) = \begin{cases} 1 & 0 \le t \le T \\ -1 & T \le t \le 2T \\ 0 & else \end{cases}$

4.1 Give a general formula for computing the output function g(t)

4.2 Plot f(s) and $h(t-s)$

4.3 Compute g(t) in the interval of $T \le t \le 2T$

Signals / Systems and
Linear Systems and Control (Admission Year 2008)

Exam: Part C2 „ Linear Systems and Control “

All answers are to be reasoned!

Allowed materials:

- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time: Part C1 and Part c2 together 40 minutes

4 Questions (Part C2)

Name:………………………………………………… **Register Nr.** : ……………………….

Hints:

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Control I

Problem 1:

Consider an engineering process as defined by the following differential equation:

$$\ddot{x}(t) + 2\dot{x}(t) + x(t) = z(t) - y(t)$$

in which, *y(t)* is the controlled input, *z(t)* is the disturbance input, and *x(t)* is the output.

a) **Plot the output of the system for unit step noise *z(t) =1(t),* and zero input *y(t)=0,* in which unit step input is defined as:** $1(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$

b) **Assume that the system is controlled by the following control law** $y(t) = 4\dot{x}(t)$**. Plot the changes that will happen to the above unit input noise command.**

c) **What is the effect of the control law** $y(t) = 3x(t)$**, compared to the cases a, and b.**

d) **How would you suggest to change the control law in case c, to track any disturbance input *z(t)=w(t).***

Problem 2:

Consider an engineering process as defined by the following differential equation:

$$2\dot{x}(t) + x(t) = \dot{y}(t) - y(t)$$

Which is controlled by $y(t) = -k_r x(t); \quad -\infty < k_r < \infty$**.**

a) **Find the process transfer function** $F(s) = \dfrac{x(s)}{y(s)}$**.**

b) **Plot the open loop** $F(j\omega)$ **in the complex plane for** $k_r = 1$**.**

Hint: Find $F(j\omega)$ **for** $\omega = 0$ **and** $\omega \to \infty$.

c) **Find the number of closed loop unstable poles with respect to** k_r

d) **For what value of** k_r **the closed loop system becomes stable.**

Control II

Problem 3:

Consider a system defined by the following transfer function ($x(s)$ is the output and $y(s)$ is the input):

$$F(s) = \frac{x(s)}{y(s)} = \frac{s-1}{s+2}$$

a) Plot the root locus of the closed loop system for a pure gain controller $F_r(s) = -k_r$ for positive gains $k_r > 0$, and find the stabilizing values of k_r.

b) Plot the root locus of the closed loop system for a dynamic controller $F_r(s) = -\frac{k_r}{s}$ for positive gains $k_r > 0$.

c) Is it possible to stabilize the system with the dynamic controller in b?

Problem 4:

The state space representation of a system is given as following: ($u(t)$ is the input and $y(t)$ is the output):

$$\begin{cases} \dot{x}(t) = Ax(t) + bu(t) \\ y(t) = c^T x(t) \end{cases}; \ A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, c^T = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

a) Is the system stable?

b) Is the system completely controllable?

c) Is the system completely observable?

Find the state feedback controller gains $u(t) = [k_1 \quad k_2]x(t)$, such that the closed loop poles are assigned at $\lambda_1 = \lambda_2 = -3$.

Good Luck